

RUHR-UNIVERSITÄT BOCHUM

The Subset-Sum Problem

cryptanalysis employing a probabilistic approach

@ KU Leuven, 22.09.2016 – 13.30

Matthias Minihold

ECRYPT-NET Early Stage Researcher

Cryptology and IT-Security, Ruhr-Universität Bochum

1 The Subset-Sum problem

- Motivation
- Historical Remarks
- Easy and Hard Instances
- Evolution of Algorithms
- Technique 1 - Meet in the Middle
- Technique 2 - Enlarge Number Set
- Applications

Outline

1 The Subset-Sum problem

- Motivation
- Historical Remarks
- Easy and Hard Instances
- Evolution of Algorithms
- Technique 1 - Meet in the Middle
- Technique 2 - Enlarge Number Set
- Applications

Motivation — informal

Motivation — informal

Informally: Pack a knapsack with items to meet a weight constraint.

Motivation — informal

Informally: Pack a knapsack with items to meet a weight constraint.
Suppose you are at the airport:

- ▶ The luggage may weight $\leq S$ [kg] at check-in.



Motivation — informal

Informally: Pack a knapsack with items to meet a weight constraint.
Suppose you are at the airport:

- ▶ The luggage may weight $\leq S$ [kg] at check-in.
- ▶ Not squandering, the bag WILL weight S [kg].



Motivation — informal

Informally: Pack a knapsack with items to meet a weight constraint.
Suppose you are at the airport:

- ▶ The luggage may weight $\leq S$ [kg] at check-in.
- ▶ Not squandering, the bag WILL weight S [kg].
- ▶ n equally beautiful items with weights a_1, a_2, \dots, a_n .



Motivation — informal

Informally: Pack a knapsack with items to meet a weight constraint.
Suppose you are at the airport:

- ▶ The luggage may weight $\leq S$ [kg] at check-in.
- ▶ Not squandering, the bag WILL weight S [kg].
- ▶ n equally beautiful items with weights a_1, a_2, \dots, a_n .



Motivation — informal

Informally: Pack a knapsack with items to meet a weight constraint.
Suppose you are at the airport:

- ▶ The luggage may weight $\leq S$ [kg] at check-in.
- ▶ Not squandering, the bag WILL weight S [kg].
- ▶ n equally beautiful items with weights a_1, a_2, \dots, a_n .



Definition 1 (Subset-Sum)

Given $n, S, a_1, a_2, \dots, a_n \in \mathbb{N}$, find $I \subseteq [n] : \sum_{i \in I} a_i = S$. (1)

Outline

1 The Subset-Sum problem

- Motivation
- Historical Remarks
- Easy and Hard Instances
- Evolution of Algorithms
- Technique 1 - Meet in the Middle
- Technique 2 - Enlarge Number Set
- Applications

Historical Remarks

- ▶ Names: (0-1-)Knapsack, Subset-Sum problem

Historical Remarks

- ▶ Names: (0-1-)Knapsack, Subset-Sum problem
- ▶ First studied in 1897

Historical Remarks

- ▶ Names: (0-1-)Knapsack, Subset-Sum problem
- ▶ First studied in 1897
- ▶ Early recognized to be \mathcal{NP} -complete by reduction to 3-SAT:

Historical Remarks

- ▶ Names: (0-1-)Knapsack, Subset-Sum problem
- ▶ First studied in 1897
- ▶ Early recognized to be \mathcal{NP} -complete by reduction to 3-SAT:
 - Satisfiability with at most 3 literals per clause ($\hat{=}$ 3-SAT)

Historical Remarks

- ▶ Names: (0-1-)Knapsack, Subset-Sum problem
- ▶ First studied in 1897
- ▶ Early recognized to be \mathcal{NP} -complete by reduction to 3-SAT:
 - Satisfiability with at most 3 literals per clause ($\hat{=}$ 3-SAT)
 - \Rightarrow_R Chromatic number (= Graph Coloring)

Historical Remarks

- ▶ Names: (0-1-)Knapsack, Subset-Sum problem
- ▶ First studied in 1897
- ▶ Early recognized to be \mathcal{NP} -complete by reduction to 3-SAT:
 - Satisfiability with at most 3 literals per clause ($\hat{=}$ 3-SAT)
 - \Rightarrow_R Chromatic number (= Graph Coloring)
 - \Rightarrow_R Exact cover

Historical Remarks

- ▶ Names: (0-1-)Knapsack, Subset-Sum problem
- ▶ First studied in 1897
- ▶ Early recognized to be \mathcal{NP} -complete by reduction to 3-SAT:
 - Satisfiability with at most 3 literals per clause ($\hat{=}$ 3-SAT)
 - \Rightarrow_R Chromatic number (= Graph Coloring)
 - \Rightarrow_R Exact cover
 - \Rightarrow_R Knapsack

Historical Remarks

- ▶ Names: (0-1-)Knapsack, Subset-Sum problem
- ▶ First studied in 1897
- ▶ Early recognized to be \mathcal{NP} -complete by reduction to 3-SAT:
 - Satisfiability with at most 3 literals per clause ($\hat{=}$ 3-SAT)
 - \Rightarrow_R Chromatic number (= Graph Coloring)
 - \Rightarrow_R Exact cover
 - \Rightarrow_R Knapsack
 - worst-case instances are computationally intractable

Outline

1 The Subset-Sum problem

- Motivation
- Historical Remarks
- **Easy and Hard Instances**
- Evolution of Algorithms
- Technique 1 - Meet in the Middle
- Technique 2 - Enlarge Number Set
- Applications

Easy and Hard Instances

Given an instance by $n, S, a_1, a_2, \dots, a_n \in \mathbb{N}$ with density $d := \frac{n}{\log(\max_i a_i)}$.

Easy and Hard Instances

Given an instance by $n, S, a_1, a_2, \dots, a_n \in \mathbb{N}$ with density $d := \frac{n}{\log(\max_i a_i)}$.

Easy instances:

- ▶ ($d < 0.9408$) Solve Shortest Vector Problem in $\mathcal{L}(B)$.

Easy and Hard Instances

Given an instance by $n, S, a_1, a_2, \dots, a_n \in \mathbb{N}$ with density $d := \frac{n}{\log(\max_i a_i)}$.

Easy instances:

- ▶ ($d < 0.9408$) Solve Shortest Vector Problem in $\mathcal{L}(B)$.
- ▶ ($d > 1$) Solve Dynamic Programming Problem

Easy and Hard Instances

Given an instance by $n, S, a_1, a_2, \dots, a_n \in \mathbb{N}$ with density $d := \frac{n}{\log(\max_i a_i)}$.

Easy instances:

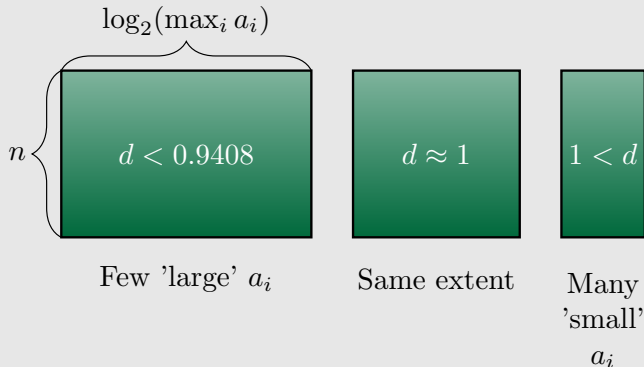
- ▶ ($d < 0.9408$) Solve Shortest Vector Problem in $\mathcal{L}(B)$.
- ▶ ($d > 1$) Solve Dynamic Programming Problem

Easy and Hard Instances

Given an instance by $n, S, a_1, a_2, \dots, a_n \in \mathbb{N}$ with density $d := \frac{n}{\log_2(\max_i a_i)}$.

Easy instances:

- ▶ ($d < 0.9408$) Solve Shortest Vector Problem in $\mathcal{L}(B)$.
- ▶ ($d > 1$) Solve Dynamic Programming Problem



Easy vs. Hard instances

Hard instances:

- ▶ Element size: $\log_2 a_i \approx n$ for $i = 1, 2, \dots, n$

Easy vs. Hard instances

Hard instances:

- ▶ Element size: $\log_2 a_i \approx n$ for $i = 1, 2, \dots, n$
- ▶ Characterized by $0.9408 < d = \frac{n}{\log(\max_i a_i)} < 1$

Easy vs. Hard instances

Hard instances:

- ▶ Element size: $\log_2 a_i \approx n$ for $i = 1, 2, \dots, n$
- ▶ Characterized by $0.9408 < d = \frac{n}{\log(\max_i a_i)} < 1$
- ▶ Balanced solution: $|I| \approx \frac{n}{2}$

Easy vs. Hard instances

Hard instances:

- ▶ Element size: $\log_2 a_i \approx n$ for $i = 1, 2, \dots, n$
- ▶ Characterized by $0.9408 < d = \frac{n}{\log(\max_i a_i)} < 1$
- ▶ Balanced solution: $|I| \approx \frac{n}{2}$
- ▶ How to generate hard instances parametrized by n :

Easy vs. Hard instances

Hard instances:

- ▶ Element size: $\log_2 a_i \approx n$ for $i = 1, 2, \dots, n$
- ▶ Characterized by $0.9408 < d = \frac{n}{\log(\max_i a_i)} < 1$
- ▶ Balanced solution: $|I| \approx \frac{n}{2}$
- ▶ How to generate hard instances parametrized by n :
 1. Choose $a \in_R \{0, 1, \dots, 2^n - 1\}^n, x \in_R \{0, 1\}^n$ uniformly.

Easy vs. Hard instances

Hard instances:

- ▶ Element size: $\log_2 a_i \approx n$ for $i = 1, 2, \dots, n$
- ▶ Characterized by $0.9408 < d = \frac{n}{\log(\max_i a_i)} < 1$
- ▶ Balanced solution: $|I| \approx \frac{n}{2}$
- ▶ How to generate hard instances parametrized by n :
 1. Choose $a \in_R \{0, 1, \dots, 2^n - 1\}^n, x \in_R \{0, 1\}^n$ uniformly.
 2. Output $(a = (a_1, a_2, \dots, a_n), S = a \cdot x = \sum_{i=1}^n x_i a_i)$.

Easy vs. Hard instances

Hard instances:

- ▶ Element size: $\log_2 a_i \approx n$ for $i = 1, 2, \dots, n$
- ▶ Characterized by $0.9408 < d = \frac{n}{\log(\max_i a_i)} < 1$
- ▶ Balanced solution: $|I| \approx \frac{n}{2}$
- ▶ How to generate hard instances parametrized by n :
 1. Choose $a \in_R \{0, 1, \dots, 2^n - 1\}^n, x \in_R \{0, 1\}^n$ uniformly.
 2. Output $(a = (a_1, a_2, \dots, a_n), S = a \cdot x = \sum_{i=1}^n x_i a_i)$.
 3. Finding x given (a, S) is (assumed to be) computationally hard.

Outline

1 The Subset-Sum problem

- Motivation
- Historical Remarks
- Easy and Hard Instances
- **Evolution of Algorithms**
- Technique 1 - Meet in the Middle
- Technique 2 - Enlarge Number Set
- Applications

Evolution of Algorithms (giving exact solutions)

Algorithm (year)	Time	Space
------------------	------	-------

Table: Expected time and space requirements of algorithms solving Eq. (1).

Evolution of Algorithms (giving exact solutions)

Algorithm (year)	Time	Space
Exhaustive Search	$2^{1.000n} \approx (2.000)^n$	$2^{0.000n} \approx (1.000)^n$

Table: Expected time and space requirements of algorithms solving Eq. (1).

Algorithm (year)	Time	Space
Exhaustive Search > Idea: <i>'meet-in-the-middle'</i>	$2^{1.000n} \approx (2.000)^n$	$2^{0.000n} \approx (1.000)^n$

Table: Expected time and space requirements of algorithms solving Eq. (1).

Algorithm (year)	Time	Space
Exhaustive Search > Idea: <i>'meet-in-the-middle'</i>	$2^{1.000n} \approx (2.000)^n$	$2^{0.000n} \approx (1.000)^n$
Horowitz-Sahni (1974)	$2^{0.500n} \approx (1.414)^n$	$2^{0.500n} \approx (1.414)^n$

Table: Expected time and space requirements of algorithms solving Eq. (1).

Algorithm (year)	Time	Space
Exhaustive Search > Idea: <i>'meet-in-the-middle'</i>	$2^{1.000n} \approx (2.000)^n$	$2^{0.000n} \approx (1.000)^n$
Horowitz-Sahni (1974) > Idea: <i>'4-list approach'</i>	$2^{0.500n} \approx (1.414)^n$	$2^{0.500n} \approx (1.414)^n$

Table: Expected time and space requirements of algorithms solving Eq. (1).

Algorithm (year)	Time	Space
Exhaustive Search > Idea: <i>'meet-in-the-middle'</i>	$2^{1.000n} \approx (2.000)^n$	$2^{0.000n} \approx (1.000)^n$
Horowitz-Sahni (1974) > Idea: <i>'4-list approach'</i>	$2^{0.500n} \approx (1.414)^n$	$2^{0.500n} \approx (1.414)^n$
Schroepel-Shamir (1979)	$2^{0.500n} \approx (1.414)^n$	$2^{0.250n} \approx (1.189)^n$

Table: Expected time and space requirements of algorithms solving Eq. (1).

Algorithm (year)	Time	Space
Exhaustive Search > Idea: <i>'meet-in-the-middle'</i>	$2^{1.000n} \approx (2.000)^n$	$2^{0.000n} \approx (1.000)^n$
Horowitz-Sahni (1974) > Idea: <i>'4-list approach'</i>	$2^{0.500n} \approx (1.414)^n$	$2^{0.500n} \approx (1.414)^n$
Schroepel-Shamir (1979) > Idea: <i>'multiple representations'</i>	$2^{0.500n} \approx (1.414)^n$	$2^{0.250n} \approx (1.189)^n$

Table: Expected time and space requirements of algorithms solving Eq. (1).

Algorithm (year)	Time	Space
Exhaustive Search > Idea: <i>'meet-in-the-middle'</i>	$2^{1.000n} \approx (2.000)^n$	$2^{0.000n} \approx (1.000)^n$
Horowitz-Sahni (1974) > Idea: <i>'4-list approach'</i>	$2^{0.500n} \approx (1.414)^n$	$2^{0.500n} \approx (1.414)^n$
Schroeppel-Shamir (1979) > Idea: <i>'multiple representations'</i>	$2^{0.500n} \approx (1.414)^n$	$2^{0.250n} \approx (1.189)^n$
Howgrave-Graham-Joux (2010) > Idea: <i>'enlarged number-set'</i> $\{-1, 0, 1\}$	$2^{0.337n} \approx (1.263)^n$	$2^{0.311n} \approx (1.241)^n$

Table: Expected time and space requirements of algorithms solving Eq. (1).

Algorithm (year)	Time	Space
Exhaustive Search > Idea: <i>'meet-in-the-middle'</i>	$2^{1.000n} \approx (2.000)^n$	$2^{0.000n} \approx (1.000)^n$
Horowitz-Sahni (1974) > Idea: <i>'4-list approach'</i>	$2^{0.500n} \approx (1.414)^n$	$2^{0.500n} \approx (1.414)^n$
Schroeppel-Shamir (1979) > Idea: <i>'multiple representations'</i>	$2^{0.500n} \approx (1.414)^n$	$2^{0.250n} \approx (1.189)^n$
Howgrave-Graham-Joux (2010) > Idea: <i>'enlarged number-set'</i> $\{-1, 0, 1\}$	$2^{0.337n} \approx (1.263)^n$	$2^{0.311n} \approx (1.241)^n$
Becker-Coron-Joux (2011)	$2^{0.291n} \approx (1.223)^n$	$2^{0.291n} \approx (1.223)^n$

Table: Expected time and space requirements of algorithms solving Eq. (1).

Algorithm (year)	Time	Space
Exhaustive Search > Idea: <i>'meet-in-the-middle'</i>	$2^{1.000n} \approx (2.000)^n$	$2^{0.000n} \approx (1.000)^n$
Horowitz-Sahni (1974) > Idea: <i>'4-list approach'</i>	$2^{0.500n} \approx (1.414)^n$	$2^{0.500n} \approx (1.414)^n$
Schroeppel-Shamir (1979) > Idea: <i>'multiple representations'</i>	$2^{0.500n} \approx (1.414)^n$	$2^{0.250n} \approx (1.189)^n$
Howgrave-Graham-Joux (2010) > Idea: <i>'enlarged number-set'</i> $\{-1, 0, 1\}$	$2^{0.337n} \approx (1.263)^n$	$2^{0.311n} \approx (1.241)^n$
Becker-Coron-Joux (2011) > Idea: <i>'quantum algorithm'</i>	$2^{0.291n} \approx (1.223)^n$	$2^{0.291n} \approx (1.223)^n$

Table: Expected time and space requirements of algorithms solving Eq. (1).

Evolution of Algorithms (giving exact solutions)

Algorithm (year)	Time	Space
Exhaustive Search > Idea: <i>'meet-in-the-middle'</i>	$2^{1.000n} \approx (2.000)^n$	$2^{0.000n} \approx (1.000)^n$
Horowitz-Sahni (1974) > Idea: <i>'4-list approach'</i>	$2^{0.500n} \approx (1.414)^n$	$2^{0.500n} \approx (1.414)^n$
Schroeppel-Shamir (1979) > Idea: <i>'multiple representations'</i>	$2^{0.500n} \approx (1.414)^n$	$2^{0.250n} \approx (1.189)^n$
Howgrave-Graham-Joux (2010) > Idea: <i>'enlarged number-set'</i> $\{-1, 0, 1\}$	$2^{0.337n} \approx (1.263)^n$	$2^{0.311n} \approx (1.241)^n$
Becker-Coron-Joux (2011) > Idea: <i>'quantum algorithm'</i>	$2^{0.291n} \approx (1.223)^n$	$2^{0.291n} \approx (1.223)^n$
Bernstein-Jeffery-Lange-Meurer (2013)	$2^{0.241n} \approx (1.182)^n$	$2^{0.241n} \approx (1.182)^n$

Table: Expected time and space requirements of algorithms solving Eq. (1).

- ▶ Asymptotically best exact algorithm is a quantum algorithm

Algorithm (year)	Time	Space
Exhaustive Search > Idea: <i>'meet-in-the-middle'</i>	$2^{1.000n} \approx (2.000)^n$	$2^{0.000n} \approx (1.000)^n$
Horowitz-Sahni (1974) > Idea: <i>'4-list approach'</i>	$2^{0.500n} \approx (1.414)^n$	$2^{0.500n} \approx (1.414)^n$
Schroeppel-Shamir (1979) > Idea: <i>'multiple representations'</i>	$2^{0.500n} \approx (1.414)^n$	$2^{0.250n} \approx (1.189)^n$
Howgrave-Graham-Joux (2010) > Idea: <i>'enlarged number-set'</i> $\{-1, 0, 1\}$	$2^{0.337n} \approx (1.263)^n$	$2^{0.311n} \approx (1.241)^n$
Becker-Coron-Joux (2011) > Idea: <i>'quantum algorithm'</i>	$2^{0.291n} \approx (1.223)^n$	$2^{0.291n} \approx (1.223)^n$
Bernstein-Jeffery-Lange-Meurer (2013)	$2^{0.241n} \approx (1.182)^n$	$2^{0.241n} \approx (1.182)^n$

Table: Expected time and space requirements of algorithms solving Eq. (1).

- ▶ Asymptotically best exact algorithm is a quantum algorithm
- ▶ Lower bound on running time is unknown

Outline

1 The Subset-Sum problem

- Motivation
- Historical Remarks
- Easy and Hard Instances
- Evolution of Algorithms
- **Technique 1 - Meet in the Middle**
- Technique 2 - Enlarge Number Set
- Applications

Technique 1 - Meet in the Middle

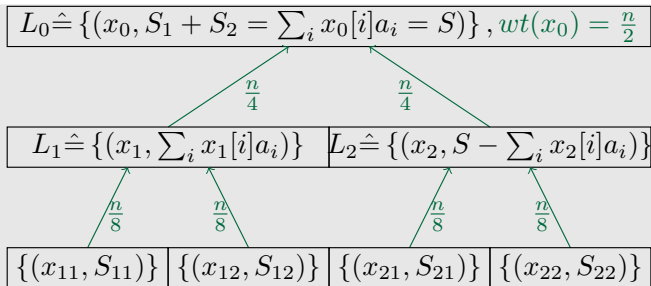


Figure: Schröppel-Shamir: Combining disjoint sub-problems of smaller weight.

Technique 1 - Meet in the Middle

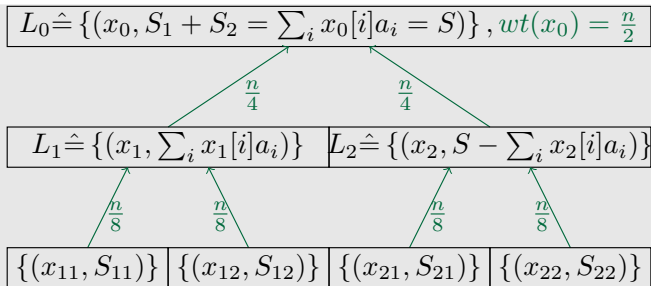


Figure: Schröppel-Shamir: Combining disjoint sub-problems of smaller weight.

- Identify subsets $I \subseteq [n]$ with vectors $x \in \{0, 1\}^n : i \in I \Leftrightarrow x[i] = 1$

Technique 1 - Meet in the Middle

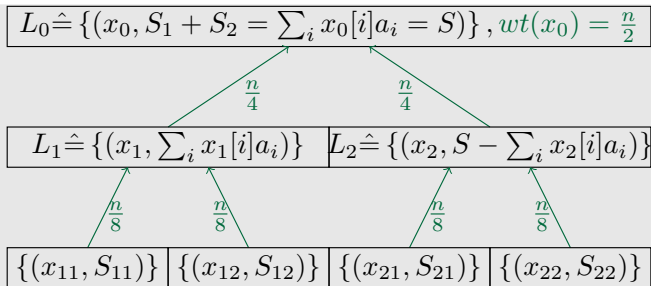


Figure: Schröppel-Shamir: Combining disjoint sub-problems of smaller weight.

- ▶ Identify subsets $I \subseteq [n]$ with vectors $x \in \{0, 1\}^n : i \in I \Leftrightarrow x[i] = 1$
- ▶ Construct collisions based on the birthday-paradox

Technique 1 - Meet in the Middle

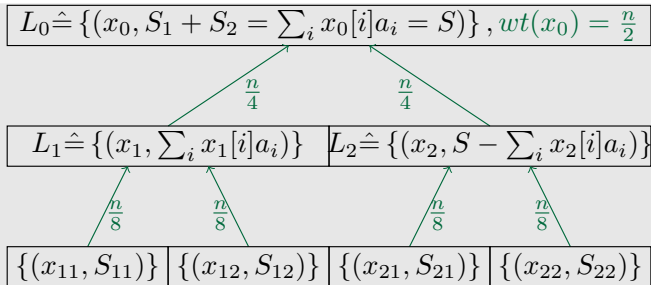


Figure: Schröppel-Shamir: Combining disjoint sub-problems of smaller weight.

- ▶ Identify subsets $I \subseteq [n]$ with vectors $x \in \{0, 1\}^n : i \in I \Leftrightarrow x[i] = 1$
- ▶ Construct collisions based on the birthday-paradox
- ▶ Construct lists L_1, L_2 of (vector, sum)-pairs in smaller dimension

Technique 1 - Meet in the Middle

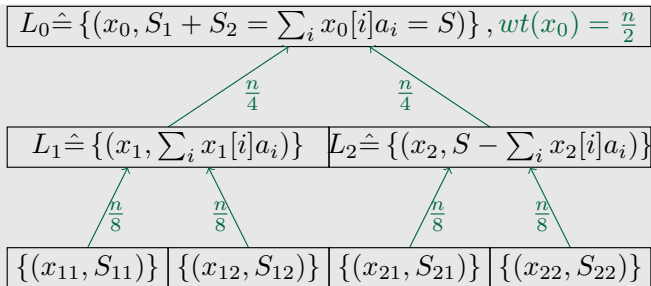


Figure: Schröppel-Shamir: Combining disjoint sub-problems of smaller weight.

- ▶ Identify subsets $I \subseteq [n]$ with vectors $x \in \{0, 1\}^n : i \in I \Leftrightarrow x[i] = 1$
- ▶ Construct collisions based on the birthday-paradox
- ▶ Construct lists L_1, L_2 of (vector, sum)-pairs in smaller dimension
- ▶ Merge L_1, L_2 to solutions $(x_0, S) \in L_0$ of the Subset-Sum problem

Outline

- 1** The Subset-Sum problem
 - Motivation
 - Historical Remarks
 - Easy and Hard Instances
 - Evolution of Algorithms
 - Technique 1 - Meet in the Middle
 - Technique 2 - Enlarge Number Set**
 - Applications

Technique 2 - Enlarge Number Set

$$L_0 \hat{=} \{(x_0, \sum_i x_0[i]a_i = S_1 + S_2 = S)\}, wt(x_0) = \frac{n}{2}$$

$$=$$

$$L_1 \hat{=} \{(x_1, \sum_i x_1[i]a_i)\}, wt(x_1) = \frac{n}{4}$$

$$+$$

$$L_2 \hat{=} \{(x_2, S - \sum_i x_2[i]a_i)\}, wt(x_2) = \frac{n}{4}$$

Figure: Adding length n solutions of sub-problems enlarges the number-set.

Technique 2 - Enlarge Number Set

$$\begin{aligned}
 & L_0 \hat{=} \{(x_0, \sum_i x_0[i]a_i = S_1 + S_2 = S)\}, wt(x_0) = \frac{n}{2} \\
 & \qquad \qquad \qquad = \\
 & L_1 \hat{=} \{(x_1, \sum_i x_1[i]a_i)\}, wt(x_1) = \frac{n}{4} \\
 & \qquad \qquad \qquad + \\
 & L_2 \hat{=} \{(x_2, S - \sum_i x_2[i]a_i)\}, wt(x_2) = \frac{n}{4}
 \end{aligned}$$

Figure: Adding length n solutions of sub-problems enlarges the number-set.

- ▶ Merge non-disjoint partial solutions in L_1, L_2 filtering 'inconsistent'

Technique 2 - Enlarge Number Set

$$\begin{aligned}
 & L_0 \hat{=} \{(x_0, \sum_i x_0[i]a_i = S_1 + S_2 = S)\}, wt(x_0) = \frac{n}{2} \\
 & \qquad \qquad \qquad = \\
 & L_1 \hat{=} \{(x_1, \sum_i x_1[i]a_i)\}, wt(x_1) = \frac{n}{4} \\
 & \qquad \qquad \qquad + \\
 & L_2 \hat{=} \{(x_2, S - \sum_i x_2[i]a_i)\}, wt(x_2) = \frac{n}{4}
 \end{aligned}$$

Figure: Adding length n solutions of sub-problems enlarges the number-set.

- ▶ Merge non-disjoint partial solutions in L_1, L_2 filtering 'inconsistent'
- ▶ Enlarge intermediate 'number-set': $x_{11}[i] + x_{12}[i] =: x_1[i] \notin \{0, 1\}$.

Technique 2 - Enlarge Number Set

$$\begin{aligned}
 & L_0 \hat{=} \{(x_0, \sum_i x_0[i] a_i = S_1 + S_2 = S)\}, wt(x_0) = \frac{n}{2} \\
 & \qquad \qquad \qquad = \\
 & L_1 \hat{=} \{(x_1, \sum_i x_1[i] a_i)\}, wt(x_1) = \frac{n}{4} \\
 & \qquad \qquad \qquad + \\
 & L_2 \hat{=} \{(x_2, S - \sum_i x_2[i] a_i)\}, wt(x_2) = \frac{n}{4}
 \end{aligned}$$

Figure: Adding length n solutions of sub-problems enlarges the number-set.

- ▶ Merge non-disjoint partial solutions in L_1, L_2 filtering 'inconsistent'
- ▶ Enlarge intermediate 'number-set': $x_{11}[i] + x_{12}[i] =: x_1[i] \notin \{0, 1\}$.
- ▶ Speed-ups if $x_1[i], x_2[i] \sim \mathcal{D}(\{-1, 0, 1\})$ w.r.t. distribution \mathcal{D} .

Outline

1 The Subset-Sum problem

- Motivation
- Historical Remarks
- Easy and Hard Instances
- Evolution of Algorithms
- Technique 1 - Meet in the Middle
- Technique 2 - Enlarge Number Set
- Applications

- ▶ These cryptanalytic methods are meta-techniques

- ▶ These cryptanalytic methods are meta-techniques
- ▶ Applicable to lattice- or code-based \mathcal{NP} -complete problems

- ▶ These cryptanalytic methods are meta-techniques
- ▶ Applicable to lattice- or code-based \mathcal{NP} -complete problems

Example 2 (Outsourcing a database to the cloud [1])

- ▶ These cryptanalytic methods are meta-techniques
- ▶ Applicable to lattice- or code-based \mathcal{NP} -complete problems

Example 2 (Outsourcing a database to the cloud [1])

Setup: A preprocessed length n text T is stored in the cloud.

- ▶ These cryptanalytic methods are meta-techniques
- ▶ Applicable to lattice- or code-based \mathcal{NP} -complete problems

Example 2 (Outsourcing a database to the cloud [1])

Setup: A preprocessed length n text T is stored in the cloud.

Given: A pattern P of length $m \leq n$, find all locations of P in T .

- ▶ These cryptanalytic methods are meta-techniques
- ▶ Applicable to lattice- or code-based \mathcal{NP} -complete problems

Example 2 (Outsourcing a database to the cloud [1])

Setup: A preprocessed length n text T is stored in the cloud.

Given: A pattern P of length $m \leq n$, find all locations of P in T .

Goal: Cloud learns no information about text T and pattern P .



S. Faust, C. Hazay, and D. Venturi.

Outsourced pattern matching.

Cryptology ePrint Archive, Report 2014/662, 2014.

<http://eprint.iacr.org/2014/662>.



RUHR-UNIVERSITÄT BOCHUM

QUESTIONS?

Thank you for your attention!